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# Turbulent Kinetic Energy Budget for MYNN-EDMF PBL Scheme in WRF model

Franciano Scremin Puhales[1a,1b,*], Joseph B. Olson[2], Jimy Dudhia[3], Douglas Lima de Bem[1a], Rafael Maroneze[1b,4], Otávio Costa Acevedo[1b], Felipe Denardin Costa[1b,4], and Vagner Anabor[1a]

[1a]Universidade Federal de Santa Maria (UFSM) – Grupo de Turbulência Atmosférica, Santa Maria, Rio Grande do Sul, Brazil
[1b]Universidade Federal de Santa Maria (UFSM) – Grupo de Modelagem Atmosférica, Santa Maria, Rio Grande do Sul, Brazil
[2]National Oceanic and Atmospheric Administration (NOAA) – Earth System Research Laboratory, Boulder, Colorado, US
[3]National Center for Atmospheric Research (NCAR) – Mesoscale & Microscale Meteorology Laboratory, Boulder, Colorado, US
[4]Universidade Federal do Pampa (Unipampa) – Laboratório de Fluidodinâmica Computacional e turbulência Atmosférica, Alegrete, Rio Grande do Sul, Brazil

[*]Corresponding author: franciano.puhales@ufsm.br

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## CONTENTS

## TECHNICAL NOTE'S AIM

The aim of this technical note is to describe the Turbulent Kinetic Energy (TKE) budget equation in Mellor-Yamada-Nakanishi-Niino (MYNN) PBL scheme of WRF model and how they are outputted. The MYNN scheme is an improved version of classic Mellor-Yamada scheme (Mellor and Yamada 1982) and it is able to output the TKE budget equation terms on the default WRF history file. These outputs were unbalanced with the equivalent terms used on MYNN to integrate the TKE budget equation. It was fixed on WRF 4.2.2 version code for tests released on WRF 4.5 (CCPP version).

Besides the TKE output fix, a new set of similarity relationship equations was implemented in the MYNN scheme to provide the lower boundary conditions for TKE budget equation terms.

## 1 MYNN-EDMF'S TKE BUDGET EQUATION

In MYNN-EDMF, the $Q$ Budget equation is given by (Nakanishi and Niino 2009; Olson et al. 2019):

$$\frac{\partial Q}{\partial t} = T_{rq} + P_{sq} + P_{bq} + D_q \tag{1.1}$$

where $Q = q^2 = 2e$ is twice the TKE $e$, and $T_{rq}$, $P_{sq}$, $P_{bq}$, and $D_q$ are the vertical transport, shear production, buoyancy production/destruction, and dissipation rates of $Q$ respectively. As $Q = 2e$, therefore:

$$\begin{aligned} \frac{\partial e}{\partial t} &= \frac{1}{2}T_{rq} + \frac{1}{2}P_{sq} + \frac{1}{2}P_{bq} + \frac{1}{2}D_q \\ \frac{\partial e}{\partial t} &= T_r + P_s + P_b + D \end{aligned} \tag{1.2}$$

where the right side of equation 1.2 has the analogous terms of 1.1 related to TKE.

Equation 1.1 is solved using an implicit time-integration method and it is discretized as follows:

$$\underbrace{\frac{Q^{n+1}_{k+1/2} - Q^{n}_{k+1/2}}{\Delta t}}_{\text{TKE tendency}} = \underbrace{\underbrace{\frac{\partial}{\partial z}\left[K^n_q \frac{\partial}{\partial z}\left(Q^{n+1}\right)\right]_{k+1/2}}_{\text{Eddy difusivity}} + \underbrace{\frac{\partial}{\partial z}\left[M^n\left(Q^{n+1} - Q^n_u\right)\right]_{k+1/2}}_{\text{Mass-flux transport}}}_{\text{TKE transport}} + \underbrace{P^n_{k+1/2}}_{\substack{\text{TKE turbulent} \\ \text{Production} \\ \text{Destruction}}} \underbrace{- F^n_{k+1/2} Q^{n+1}_{k+1/2}}_{\text{Dissipation rate}} \tag{1.3}$$

where $n$ denotes the time index and $k$ denotes the vertical level index. As the WRF grid is staggered, the full levels (cell faces) are represented by the integer $k$ index, while the half levels are represented by $k+1/2$ index levels (for further details see appendix A). In equation 1.3, $Q$ is a half level variable, while $K_q$ and $M$ are full level variables. Furthermore, the first and second terms of the right side are the eddy diffusion and mass-flux $Q$ transport terms, $P$ is the sum of shear and buoyancy terms, and the last term $FQ$ is the dissipation rate. The time-integration of equation 1.3 is solved by the following relationship:

$$aQ^{n+1}_{k-1/2} + bQ^{n+1}_{k+1/2} + cQ^{n+1}_{k+3/2} = d \tag{1.4}$$

where $a$, $b$, and $c$ are the matrix elements to solve the implicit system for $Q^{n+1}_{k-1/2}$.

To output all TKE budget equation terms in wrfout_d<domain>_AAAA-MM-DD_HH:mm files, it is necessary to use the bl_mynn_tkebudget(maxdom) variable in &physics block of namelist.input file:

$$\text{bl\_mynn\_tkebudget(maxdom)} = \begin{cases} 0, & \text{not outputted (default)} \\ 1, & \text{outputted} \end{cases}$$

The output variables associated to TKE budget equation are:

Table 1.1 – Summary of TKE budget equation variables outputted in wrfout_d<domain>_AAAA-MM-DD_HH:mm files.

| TKE Budget term | Variable name | Description |
|---|---|---|
| Mechanical production | QSHEAR | TKE Production - shear |
| Buoyancy production/destruction | QBUOY | TKE Production - buoyancy |
| TKE vertical transport | QWT | TKE vertical transport |
| TKE dissipation rate | QDISS | TKE dissipation |
| TKE tendency | DTKE | TKE tendency |

In this WRF version, all outputted TKE budget variables are obtained as a "mass" grid point variable (see appendix A for more details).

## 1.1 Production terms

The TKE production[1] terms are given by:

$$P_s = -\left\langle u'w' \right\rangle \frac{\partial u}{\partial z} - \left\langle v'w' \right\rangle \frac{\partial v}{\partial z} \tag{1.5}$$

$$P_b = \frac{g}{\theta_0} \left\langle w'\theta_v' \right\rangle \tag{1.6}$$

since $g = 9,81\ m\ s^{-2}$ is the gravity acceleration and $\theta_0 = 300\ K$ is the reference potential temperature.

The turbulent fluxes are evaluated from local-gradient and counter-gradient terms (Nakanishi and Niino 2009; Olson et al. 2019):

$$\left\langle w'\phi' \right\rangle = -K_{h,m} \left( \frac{\partial \phi}{\partial z} - \gamma \right) \tag{1.7}$$

For the momentum fluxes, $\gamma = 0$ and $K_m = \ell q S_m$ were considered, while for scalar fluxes $K_h = \ell q S_h$ was employed. Also, $\left\langle w'\theta_v' \right\rangle$ is divided in two terms, local ($L$) and a non-local (NL) (Olson et al. 2019):

$$\left\langle w'\theta_v' \right\rangle = \left\langle w'\theta_v' \right\rangle^L + \left\langle w'\theta_v' \right\rangle^{NL} \tag{1.8}$$

[1] $P_b$ is a production or destruction term depending on the static stability.

In equation 1.3, $P^n_{k+1/2}$ is given by:

$$P^n_{k+1/2} = P^n_{s_{i+1/2}} + P^n_{b_{i+1/2}} \tag{1.9}$$

$$P^n_{s_{k+1/2}} = \frac{1}{2}\left(P^n_{s_k} + P^n_{s_{k+1}}\right) \tag{1.10}$$

$$P^n_{b_{k+1/2}} = \frac{1}{2}\left(P^n_{b_k} + P^n_{b_{k+1}}\right) \tag{1.11}$$

For $k > 0$:

$$P_{s_k} = \ell_k q_k S_{m_k} g_{m_k} \tag{1.12}$$

$$g_{m_k} = \frac{\left(u_{k+1/2} - u_{k-1/2}\right)^2 - \left(v_{k+1/2} - v_{k-1/2}\right)^2}{\Delta z^2_{k-1/2}} \tag{1.13}$$

$$P_{b_k} = -\ell_k q_k \left(S_{h_k} g_{h_k} - \gamma_\theta\right) + P_{bnl_k} \tag{1.14}$$

$$g_{h_k} = \frac{\left(\theta_{v_{k+1/2}} - \theta_{v_{k-1/2}}\right)}{\Delta z_{k-1/2}} \frac{g}{\theta_0} \tag{1.15}$$

$$\tag{1.16}$$

where $P_{bnl_k}$ is the non-local TKE buoyancy production/destruction. Since $q$ is a half-level variable, $q_k$ is obtained by a linear interpolation of $q_{k-1/2}$ and $q_{k+1/2}$. Furthermore, the vertical grid spacing between full and half levels are given by:

$$\Delta z_k = z_{k+1} - z_k \tag{1.17a}$$

$$\Delta z_{k+1/2} = \frac{1}{2}\left(\Delta z_k + \Delta z_{k+1}\right) \tag{1.17b}$$

where $z_k = 0$ since $k = 0$.

Similarity relationships are used as lower boundary condition to obtain these terms for $k = 0$:

$$P_{s_k} = 2\left(\frac{u^3_*}{\kappa z_{1/2}}\right)\phi_m - P_{s_{k+1}} \tag{1.18}$$

$$P_{b_k} = 2\left(\frac{u^3_*}{\kappa z_{1/2}}\right)\zeta - P_{b_{k+1}} \tag{1.19}$$

where $u_*$ is the friction velocity, $\zeta = \dfrac{z_{1/2}}{L}$ is the stability parameter, $L$ is the Monin-Obukhov length, $\phi_m$ is a stability function, $z_{1/2} = \left(\dfrac{1}{2}\right)\Delta z_1$, and $\kappa = 0.4$ is the von Karman constant.

The MYNN has used a Kansas-type similarity gradient function to prescribe the TKE production at the first vertical half level, given by (Arya 2001):

$$\phi_{mK} = \begin{cases} (1 - a_{mK}\zeta)^{-1/4}, & \text{if } \zeta < 0 \\ (1 + b_{mK}\zeta), & \text{if } \zeta \geq 0 \end{cases} \tag{1.20}$$

where $a_{mk} = 16$, $b_{mk} = 5$, and $\zeta$ has it value updated inside MYNN with the surface heat flux obtained from LSM (land-surface model), as follows:

$$\zeta = -z_{1/2} \left( \frac{\kappa \frac{g}{\theta_0} \langle w'\theta'_v \rangle_s}{max(u_*^3, 1 \times 10^{-6})} \right) \tag{1.21}$$

A new stability function was implemented in WRF 4.2.2 as a testing option and it is described in section 2. This new option is based on Jiménez et al. (2012) stability functions for profile relationships. This new set of equations working in the same $\zeta$ domain that ones used on MM5 revised and MYNN surface layer (SFCLAY) modules, which are compatible with MYNN PBL scheme.

## 1.2 Vertical transport term

The vertical transport term is divided in an eddy diffusivity ($ED$, local) and a mass-flux ($MF$, non-local) terms:

$$T^n_{r_{k+1/2}} = ED^n_{k+1/2} + \gamma_{MF} MF^n_{k+1/2} \tag{1.22a}$$

$$ED^n_{k+1/2} = \frac{1}{2\Delta z_k} \left[ \frac{K^n_{q_{k+1}}}{\Delta z_{k+1/2}} \left( Q^{n+1}_{k+3/2} - Q^{n+1}_{k+1/2} \right) - \frac{K^n_{q_i}}{\Delta z_{k-1/2}} \left( Q^{n+1}_{k+1/2} - Q^{n+1}_{k-1/2} \right) \right] \tag{1.22b}$$

$$MF^n_{k+1/2} = \frac{1}{\Delta z_k} \left[ \left( \frac{M^n_{k+1}}{2} \right) Q^{n+1}_{k+3/2} + \left( \frac{M^n_{k+1} - M^n_k}{2} \right) Q^{n+1}_{k+1/2} - \left( \frac{M^n_k}{2} \right) Q^{n+1}_{k-1/2} + M^n_k Q^n_{u_k} - M^n_{k+1} Q^n_{u_{k+1}} \right] \tag{1.22c}$$

where $\gamma_{MF} = 0$ or 1 is a activation parameter that controls the MF term. It is changed in the namelist file trough the variable bl_mynn_edmf_tke(maxdom), as follows:

$$\text{bl\_mynn\_edmf\_tke(maxdom)} = \begin{cases} 0, & \gamma_{MF} = 0\text{: MF term is turned off (default)} \\ 1, & \gamma_{MF} = 1\text{: MF term is activated} \end{cases}$$

## 1.3 TKE dissipation rate

The TKE dissipation rate is given by:

$$\varepsilon^n_{k+1/2} = -\frac{1}{2} F^n_{k+1/2} Q^{n+1}_{k+1/2} \tag{1.23}$$

where $F^n_{k+1/2} = \dfrac{q^n_{k+1/2}}{b_1 \ell^n_{k+1/2}}$, $b_1 = 24$ (Nakanishi and Niino 2009). The mixing length is a full-level variable. At half levels, it is given by:

$$\ell^n_{k+1/2} = \frac{1}{2}\left(\ell^n_k + \ell^n_{k+1}\right) \tag{1.24}$$

Furthermore, for $k = 0$: $\ell^n_k = 0$.

## 2 NEW STABILITY FUNCTIONS FOR MYNN

The new gradient relationship is given by

$$\phi_\alpha = \begin{cases} \phi_{\alpha g} = 1 - \zeta \dfrac{\mathrm{d}\,\psi_{\alpha g}}{\mathrm{d}\,\zeta}, & \text{if } \zeta < 0 \\ \phi_{\alpha c} = 1 + a_{\alpha c} \dfrac{\zeta + \zeta^{b_{\alpha c}}\left(1+\zeta^{b_{\alpha c}}\right)^{b_{\alpha c}^{-1}-1}}{\zeta + \left(1+\zeta^{b_{\alpha c}}\right)^{b_{\alpha c}^{-1}}}, & \text{if } \zeta \geq 0 \end{cases} \tag{2.1}$$

where $\alpha = m, h$ identify the function for momentum (m) and heat (h), $\phi_{\alpha g}$ is the gradient similarity obtained from Grachev et al. (2000) expression for $\psi_{\alpha g}$. The constants $a_{\alpha c}$ and $b_{\alpha c}$ are presented in table 2.1 (Cheng and Brutsaert 2005):

Table 2.1 – Constants used in the new gradient relationship for neutral/stable regimes.

| $\alpha$ | $a_{\alpha c}$ | $b_{\alpha c}$ |
|---|---|---|
| m | 6.1 | 2.5 |
| h | 5.3 | 1.1 |

For both stability conditions, the similarity relationship profiles used by Jiménez et al. (2012) are related to gradient functions as follows:

$$\psi_\alpha(\zeta) = \int_0^\zeta \frac{1-\phi_\alpha(\zeta)}{\zeta}\,\mathrm{d}\,\zeta \tag{2.2}$$

The relationship between gradient and profile stability functions is used to obtain the first one for unstable regime ($\zeta < 0$). In this case, the profile relationship is given by (Grachev et al. 2000; Jiménez et al. 2012):

$$\psi_{\alpha g}(\zeta) = \frac{\psi_{\alpha K} + \zeta^2 \psi'_{\alpha g}}{1+\zeta^2} \tag{2.3}$$

where $\psi_{\alpha K}$ is the Kansas-type profile relationship (Arya 2001),

$$\psi_{\alpha K} = \begin{cases} 2\ln\left(\dfrac{1+x}{2}\right) + \ln\left(\dfrac{1+x^2}{2}\right) - 2\arctan(x) + \dfrac{\pi}{2}, & \alpha = m \\ 2\ln\left(\dfrac{1+x^2}{2}\right), & \alpha = h \end{cases} \tag{2.4}$$

where $x = (1 - a_{\alpha K})^{1/4}$, $a_{\alpha K} = a_{mK} = a_{hK} = 16$, and $\psi'_{\alpha g}$ is given by the following expression:

$$\psi'_{\alpha g}(\zeta) = \frac{3}{2}\ln\left(\frac{y_\alpha^2 + y_\alpha + 1}{3}\right) - \sqrt{3}\arctan\left[\frac{\sqrt{3}}{3}(2y_\alpha + 1)\right] + \frac{\sqrt{3}\pi}{3} \tag{2.5}$$

where $y_\alpha = (1 - r_\alpha \zeta)^{1/3}$, with $r_m = 10$ and $r_h = 34$.

From equation 2.3:

$$\begin{aligned} \frac{\mathrm{d}\,\psi_{\alpha g}}{\mathrm{d}\,\zeta} &= \frac{\left[\dfrac{\mathrm{d}}{\mathrm{d}\,\zeta}(\psi_{\alpha K}) + \zeta^2\psi'_{\alpha g}\left(1+\zeta^2\right) - \left(\psi_{\alpha K} + \zeta^2\psi'_{\alpha g}\right)\dfrac{\mathrm{d}}{\mathrm{d}\,\zeta}\left(1+\zeta^2\right)\right]}{\left(1+\zeta^2\right)^2} \\ \frac{\mathrm{d}\,\psi_{\alpha g}}{\mathrm{d}\,\zeta} &= \frac{\dfrac{1-\phi_{\alpha K}}{\zeta} + 2\zeta\psi'_{\alpha g} + \zeta^2\dfrac{\mathrm{d}\,\psi'_{\alpha g}}{\mathrm{d}\,\zeta}}{\left(1+\zeta^2\right)} \end{aligned} \tag{2.6}$$

where $\phi_{\alpha K}$ the Kansas-type gradient similarity relationship (equation 1.20).

Substituting the functions $f_\alpha = \frac{1}{3}\left(y_\alpha^2 + y_\alpha + 1\right)$ and $g_\alpha = \frac{\sqrt{3}}{3}(2y_\alpha + 1)$ in $\psi'_{\alpha g}$ it is possible to obtain the following relationships:

$$\psi'_{\alpha g} = \frac{3}{2}\ln(f_\alpha) - \sqrt{3}\arctan(g_\alpha) + \frac{\sqrt{3}\pi}{3} \tag{2.7a}$$

$$\frac{\mathrm{d}\,\psi'_{\alpha g}}{\mathrm{d}\,\zeta} = \frac{3}{2}\frac{1}{f_\alpha}\frac{\mathrm{d}\,f_\alpha}{\mathrm{d}\,\zeta} - \sqrt{3}\left(\frac{1}{1+g_\alpha^2}\right)\frac{\mathrm{d}\,g_\alpha}{\mathrm{d}\,\zeta} \tag{2.7b}$$

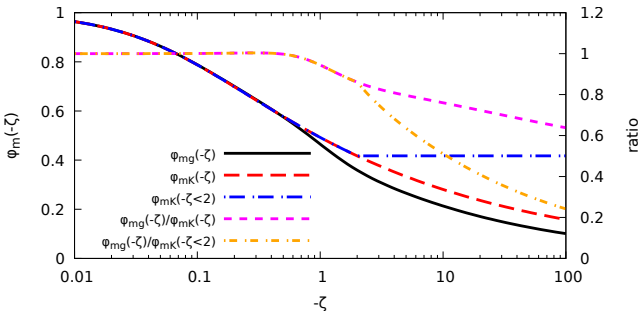
$$\frac{\mathrm{d}\,f_\alpha}{\mathrm{d}\,\zeta} = \frac{1}{3}\frac{\mathrm{d}\,y_\alpha}{\mathrm{d}\,\zeta}(2y_\alpha + 1) \tag{2.7c}$$

$$\frac{\mathrm{d}\,g_\alpha}{\mathrm{d}\,\zeta} = \frac{2\sqrt{3}}{3}\frac{\mathrm{d}\,y_\alpha}{\mathrm{d}\,\zeta} \tag{2.7d}$$

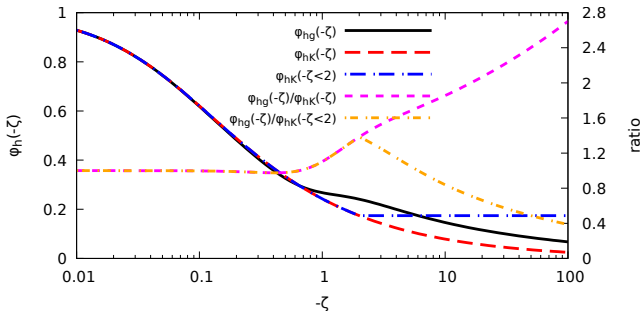
$$\frac{\mathrm{d}\,y_\alpha}{\mathrm{d}\,\zeta} = -\frac{r_\alpha}{3}(1 - r_\alpha\zeta)^{-2/3} \tag{2.7e}$$

In this way, the gradient similarity relationship for $\zeta < 0$ is obtained by substituting equations 2.5, 2.6, and 2.7 in equation 2.1.

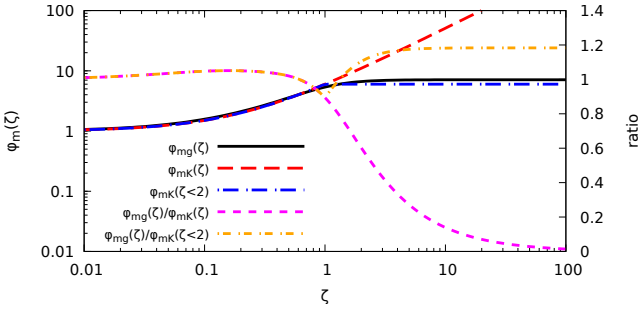
Figure 2.1 exhibits a comparison between the Kansas-type and the new gradient similarity relationships. An advantage of the new functions is the $\zeta$ domain for which $\phi_\alpha(\zeta)$ is properly defined. In general, The Kansas-type functions were obtained from experimental data interpolated in $-2 \geq \zeta \geq 1$. Thus, the new functions can be used in larger stability domain values.



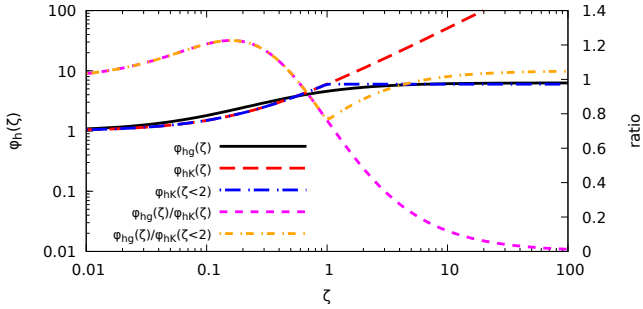

(a) $\alpha = m$ for $\zeta < 0$




(b) $\alpha = h$ for $\zeta < 0$




(c) $\alpha = m$ for $\zeta \geq 0$




(d) $\alpha = h$ for $\zeta \geq 0$

Figure 2.1

The stability function option bl_mynn_stfunc is hard-coded in WRF. It must be changed near the top of the module_bl_mynn.F before compiling, as follows:

$$\text{bl\_mynn\_stfunc} = \begin{cases} 0, & \text{Kansas-type (default)} \\ 1, & \text{new option (for testing)} \end{cases}$$

The default option is bl_mynn_stfunc = 1.

## A WRF STAGGERED GRID

The WRF grid is an Arakawa C grid where the center-cell grid points are called “mass points”. The face grid points staggered at one-half grid length from the mass points are called U, V, and W points. These faces are normal to these wind speed components (Skamarock et al. 2019). Figure A.1 shows a 3D view of a WRF grid cell where the mass and wind points are identified. The center and face points are staggered for $\frac{1}{2}\Delta x_i$, where $\Delta x_i$ is the mesh resolution in $i$ direction, the index $\alpha + 1/2$ and $\alpha - 1/2$ denotes points that are at $\pm\frac{1}{2}\Delta x_i$ from $\alpha$.

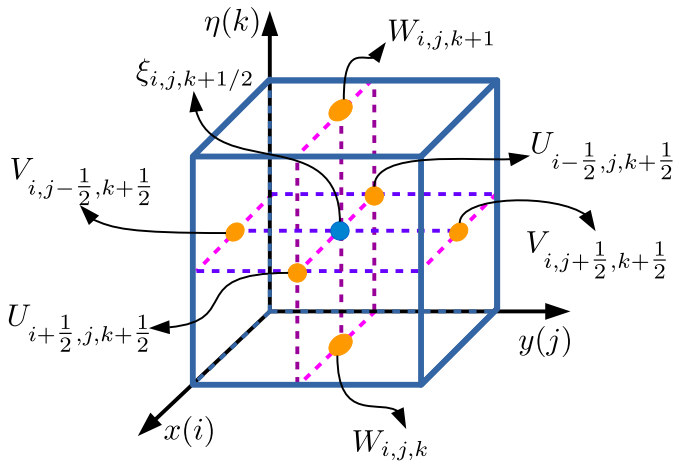



Figure A.1 – Representation of a WRF grid cell. The face-yellow dots are related to wind points while the centered-blue dot is the mass grid point.

In a vertical column of WRF grid, the TKE budget equation terms obtained on grid cell vertical faces (W points) are converted to centred point values by an average procedure. For any variable $A_k$ and $A_{k\pm1}$, the center grid point value $A_{k\pm1/2}$ is given as follows:

$$A_{k\pm\frac{1}{2}} = \frac{A_k + A_{k\pm1}}{2} \tag{A.1}$$

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